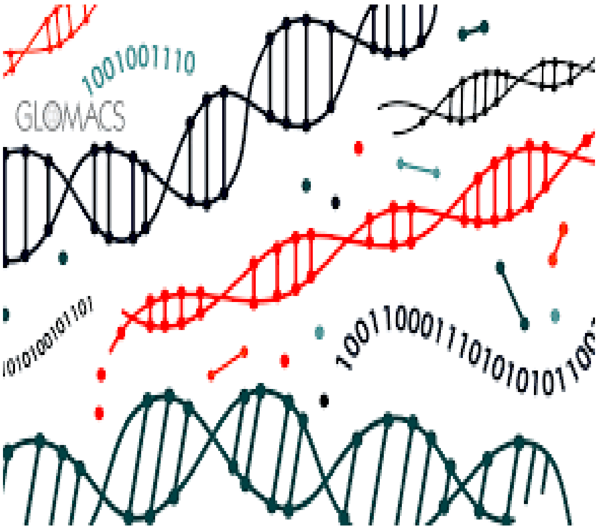
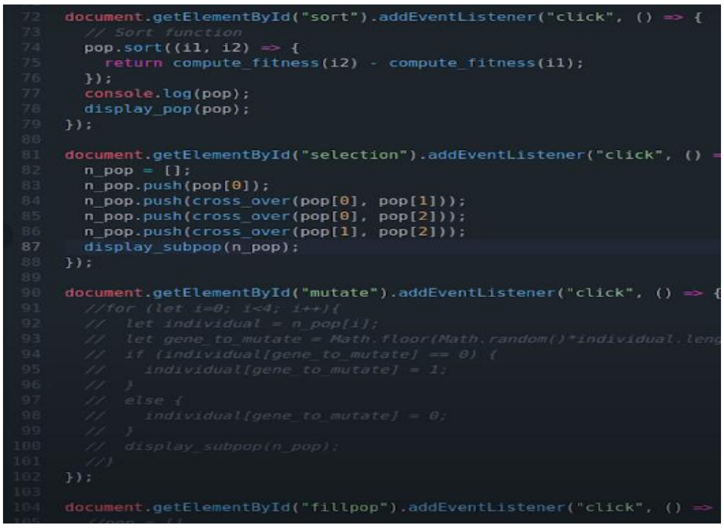
المدرسة المحمدية للمهندسين
Ecole Mohammadia d'Ingénieurs

بسم الله الرحمن الرحيم

Optimization of Cantilever Retaining Wall Using Genetic Algorithm Method





```
document.getElementById("sort").addEventListener("click", () => {
  // Sort function
  pop.sort((i1, i2) => {
    return compute_fitness(i2) - compute_fitness(i1);
  });
  console.log(pop);
  display_pop(pop);
});

document.getElementById("selection").addEventListener("click", () =
  n_pop = [];
  n_pop.push(pop[0]);
  n_pop.push(cross_over(pop[0], pop[1]));
  n_pop.push(cross_over(pop[0], pop[2]));
  n_pop.push(cross_over(pop[1], pop[2]));
  display_subpop(n_pop);
});

document.getElementById("mutate").addEventListener("click", () => {
  //for (let i=0; i<4; i++){
  //  let individual = n_pop[i];
  //  let gene_to_mutate = Math.floor(Math.random()*individual.leng
  //  if (individual[gene_to_mutate] == 0) {
  //    individual[gene_to_mutate] = 1;
  //  }
  //  else {
  //    individual[gene_to_mutate] = 0;
  //  }
  //  display_subpop(n_pop);
  //}
});

document.getElementById("fillpop").addEventListener("click", () =>
```

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1 INTRODUCTION

Retaining wall is a structural element which is intended to support back fill soils, e.g. support deep excavations, as abutments of bridge and as anchored retaining wall, etc. Retaining walls are known to be simplest structures but there design should satisfy some stability requirements. Those are internal and external stability conditions. First one talks about structural stability of different parts of retaining wall where as second one gives the wall-soil interaction after construction will be in equilibrium. While designing a structure, engineers need to adopt many technological and managerial decisions at different stages. At present the optimizing design in structural elements mainly follows rules which are formed according to experience of designer. Most of the designs will be done by assuming cross sectional dimensions and material grades and then lead to safety design by compiling them with design code requirements.

In this type of designs optimization techniques are best alternative to designs which depends on experience. Optimum design is the process of structural synthesis which give designs not only safe but economical. In any design optimization is done on reliability base or cost minimization. The constraints are formed by using stability conditions. Mathematically it is defined as minimization task of constrained function. And it is solved by a method based on simple mathematical Program. In recent years new developments in numerical methods has given some optimization techniques which are conceptually different from traditional methods. These methods are termed as non-traditional methods of optimization. These methods work depending on biological behavior, molecular swarm of insects and system of neutral biologics.

Genetic algorithm method is used as a tool in artificial intelligence and computer programming. Search is based on Darwin's theory of survival of fittest. It is non-traditional search technique in most of cases it is used to find the most economical global optimum solutions. This technique starts with a set of design points with available variables; it works on the principal of natural genetic and natural selection. From the early set of design points, new set of design points are generated and weaker points are removed.

The genetic algorithm differs from other methods in several aspects:

- Genetic algorithm works with a coding of the set of feasible point rather than the set generated itself.
- It finds from a random set of points rather than the set itself.
- Derivatives of the objective function are not used.
- It uses random operation in each iteration process.

2 Genetic Algorithm Method

2.1 Fundamentals of Genetic Algorithms

GAs are based on the principle of genetic evolution in which only the best or "fittest" individuals of a population survive to reproduce and create the next generation. During the process, some genes of the new-born individuals are randomly mutated. This operation introduces new changes that could make an individual stronger to survive in its environment, and therefore, more likely to reproduce and pass those advantages to the next generation.

In the mathematical analogy a population consist of several design vectors (individuals), each design vector is composed of a set of design variables (chromosomes) that can be modified within a specific domain or search space during the algorithm execution. In the case of single-objective optimization problems, a single objective function that assigns a fitness value to the design vectors is defined. The natural processes of selection, reproduction and mutation are simulated through numerical operations. At every iteration (or generation), the best design vectors of the population based on their fitness value are selected, combined, and mutated to create a new stronger population. The algorithm usually stops after a certain number of generations is completed, if no improvement is observed after a specific number of iterations or if another convergence criterion is satisfied.

2.2 Definitions

- **Fitness Function**

The fitness function determines how fit an individual is (the ability of an individual to compete with other individuals). It gives a fitness score to each individual. The probability that an individual will be selected for reproduction is based on its fitness score.

- **Constraint Function**

Constraint is a condition of an optimization problem that the solution must satisfy. There are several types of constraints—primarily equality constraints, inequality constraints, and integer constraints.

2.3 Genetic Algorithm with Elitism

The solution to the optimization problem is found by applying a single objective GA with elitism. The algorithm used for the present study is briefly described in the following steps:

- **Step 1. Initialization**

The algorithm starts at i = 0 by randomly generating multiple design vectors [e1, e2,. . . es] on a search space that is based on their pre-specified domain and ranges. Each design vector is denoted as an individual and all together they form the initial population Pi.

- **Step 2. Objective and constraint function evaluation**

The objective and the constraint values are computed for all the individuals of the current population Pi based on Equations.

- **Step 3. Selection of the fittest.**

The best individuals from the current population are chosen following the dominance-based tournament selection operator. This operator takes two random individuals from the current population {ea, eb} ∈ Pi to be compared or “fight” against each other.

The winner of the fight is chosen according to the following dominance criteria. If both solutions fulfil the constraint, the one with the lowest objective value dominates the other. If both solutions violate the equality constraint, the one with the lowest constraint value dominates the other.

After the winner of the first encounter is obtained, a new opponent is chosen randomly from the current population to "fight" against the previous winner. This battling process is repeated for a specific number of times known as tournament size. The winner of the final fight is the champion of the tournament, and the outcome of the selection operator.

After every tournament, the champion is removed from the available pool to avoid selecting the same individual multiple times.

- **Step 4. Crossover**

The goal of the crossover operation is to generate new elements by the reproduction of the strongest individuals. From the previously created pool Si, two individuals

{eI, eII} ∈ Si are chosen and their decision variables (chromosomes) are combined to obtain two new offspring {ex, ey} ∈ Qi. The simulated binary crossover algorithm (SBX) is used for this operation.

$$\text{crossover}(Si) = Qi$$

- **Step 5. Mutation**

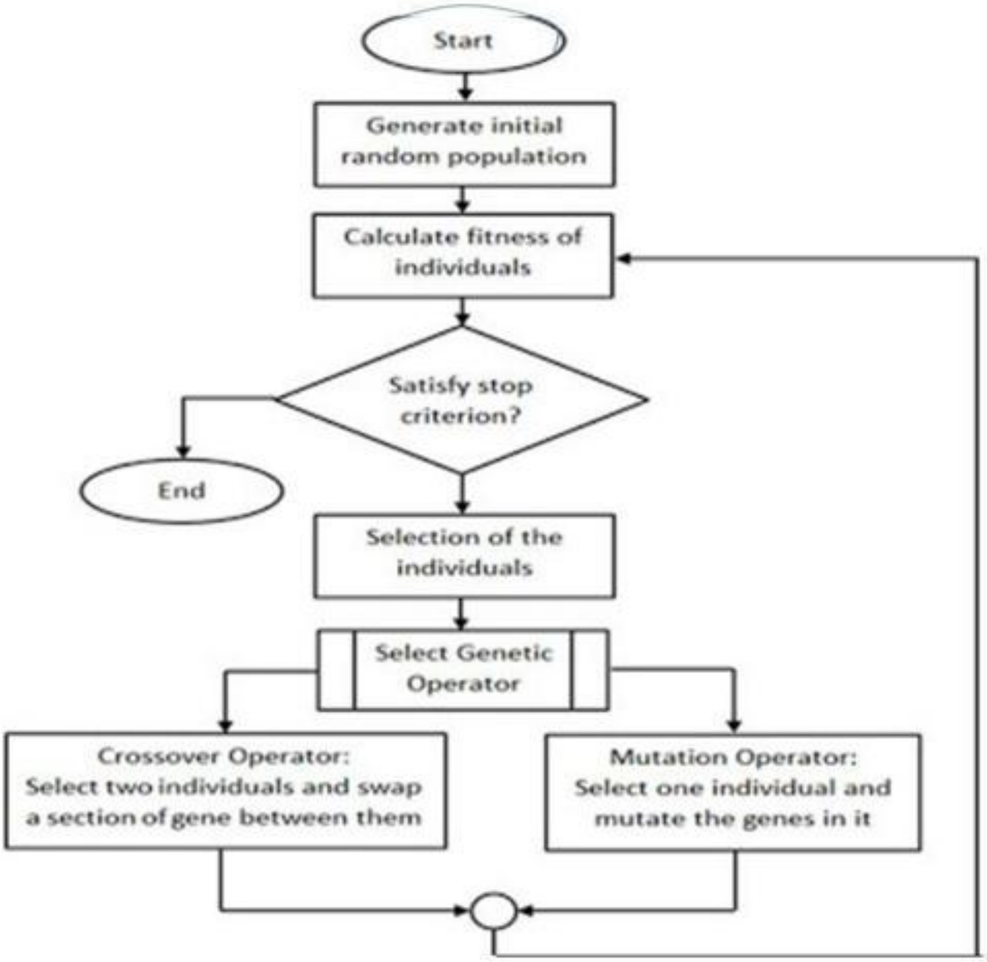
The mutation operation is introduced to the newly created population Qi. The main purpose of the mutation is to maintain diversity and avoid the problem of getting trapped into a local minimum. The mutation operator produces a small change to the decision variables using the polynomial mutation scheme. The mutation is applied with a probability of 1/n, so that on average, one variable per individual is mutated.

$$Pi+1 = \text{mutation}(Qi) \cup Si$$

Notice that elitism preservation is achieved by passing the selected elements Si unchanged to the next population.

- **Step 6. Termination**

The algorithm terminates if the population has converged (does not produce offspring which are significantly different from the previous generation). Then it is said that the genetic algorithm has provided a set of solutions to our problem.


Start
Generate initial random population
Calculate fitness of individuals
Satisfy stop criterion?
End
Selection of the individuals
Select Genetic Operator
Crossover Operator: Select two individuals and swap a section of gene between them
Mutation Operator: Select one individual and mutate the genes in it

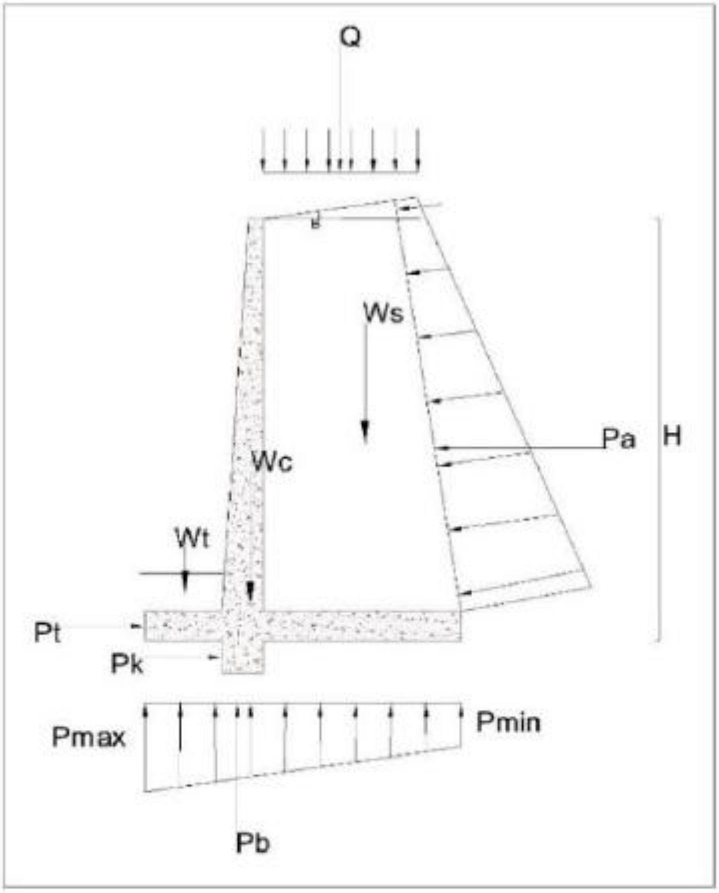
3 Study the guidelines for the design of cantilever retaining wall

3.1 General forces acting on the retaining wall

The various forces acting on the retaining wall structure are consistent; formulation includes active and passive forces on the front of the toe and base shear key sections and the bearing force of the base soil.

- Wc= combined weight of all the sections of the R.C.C wall.

- WS =weight of backfill acting on the heel.
- Wt = weight of soil on the toe.
- Q = surcharge load.
- Pa= force due to the active earth pressure. Pk and Pt =forces due to passive earth pressure on the base shear key and front part of the toe section.
- Pb =force due to the bearing stress of the base shear.



General forces acting on the retaining wall.

3.2 Design steps of retaining wall:

The following analysis and design step has been followed for the writing of MAT LAB program.

- Determination of depth of the foundation: The depth of the foundation depends upon the depth at which adequate bearing capacity is available at the site.

$$D_f = \frac{p_o}{\gamma}\left(\frac{1-\sin\phi}{1+\sin\phi}\right)$$

Where

- **po** is the safe bearing capacity of the soil at the safe at the site.
- γ is the density of the soil at the safe at the site.
- ϕ Angle of repose.

- Height of the wall: The overall height of wall H is equal to height of the backfill at the face of wall plus the depth the foundation.
- Base width: The width B of the base normally varies from 0.45H to 0.70H, depending upon the characteristics of the backfill. It is derived from the equation given below,

$$B = 1.5\sqrt{\frac{2\,(p_a\,y_a + p_s\,y_s)}{\gamma\,H}}$$

$$H = over\ all\ height\ of\ the\ retaining\ wall.$$

p_a *and* p_s *Total horizontal forces on the wall due to active earth pressure.*
y_a *and* y_s *are the heights of these forces from the base of the footing slab.*

- Stem height: The preliminary stem height is established from overall height of the wall H and the base thickness D as the thickness at the base is determined from the imposed shear and the moments.

- Design of the base slab: The heel is subjected to upward soil pressure and downward gravity loads due to its self-weight and earth above it. The latter being larger causes a hogging moments. The steel placed at the top face. The toe is subjected to the net upward pressure and reinforcement is placed at the bottom of the slab.

- Design of the stem: The stem is designed as a vertical cantilever subjected to a triangular or trapezoidal. Main reinforcement is provided on the back fill face of the stem. Secondary reinforcement may be curtailed at the top portion of the stem. Secondary reinforcement is provided @0.15% in mild steel and 0.12% in case of HYSD steel.

3.3 Stability requirement

- Overturning: To ensure stability against overturning about the edge of the toe slab, stabilizing moments due to the gravity loads should be more than the overturning moments due to lateral force so as to get a factor so as to get factor of safety in the range of 1.5 to 2.0. according to IS:456 2000,
- Sliding: The resisting force against sliding should be more than the sliding force so as get a factor of safety of 1.55

$$\text{Factor of safety} = \frac{\mu \sum W}{p_{ah}}$$

Where

$$\begin{aligned} \sum W &= Total\ gravity\ load. \\ p_{ah} &= Horizontal\ components\ of\ earth\ pressure. \\ \mu(=\tan\phi) &= friction\ coefficient\ between\ base\ and\ ground. \\ \phi &= angle\ of\ repose\ or\ angle\ of\ internal\ friction. \end{aligned}$$

In case the factor of safety is less than 1.55, a base key may be provided to obtain additional resisting force to sliding. The base key develops a passive earth pressure of

$$P_p = \frac{1}{2} k_p \gamma h_1^2 a$$

Where h1 is the depth of the base key including the thickness of the toe slab.

- Base pressure: The vertical pressure on the soil under the base should not exceed the bearing capacity of soil. There should be no tension under the base, since tension means that. The heel will be lifted above the soil, which is not permissible.

4 Optimization of cantilever retaining wall

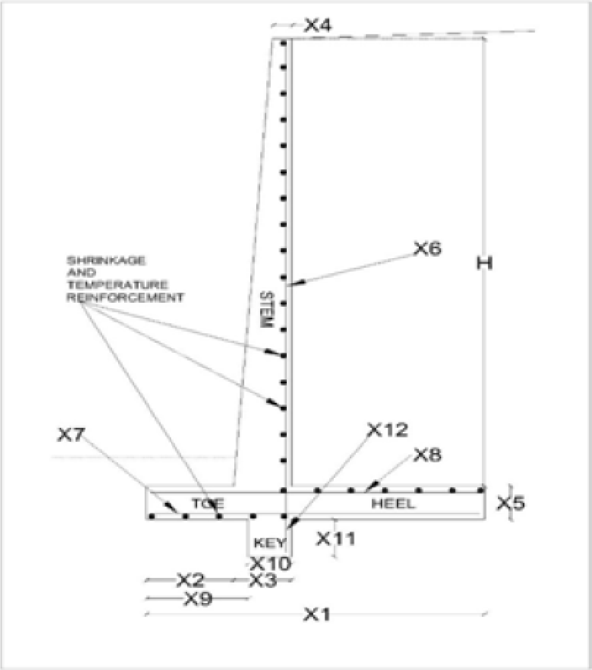
↳ Optimizer is built on bases of genetic algorithm programming; the problem is automated by writing a program in MATLAB software which is user friendly and flexible for optimization of cantilever retaining wall. Few important features of the program are mentioned bellow.

- Analysis and designing of the retaining wall is coded in the MATLAB software.
- Each commands used in the code are clearly defined in the form of comment for self-explanatory of the program.
- The user has to enter variables, constraints and the number of individuals per subpopulations and maximal Number of generations and generation gap.
- The user has to enter permissible stresses and permissible deflection for the grade of concrete selected as per code.

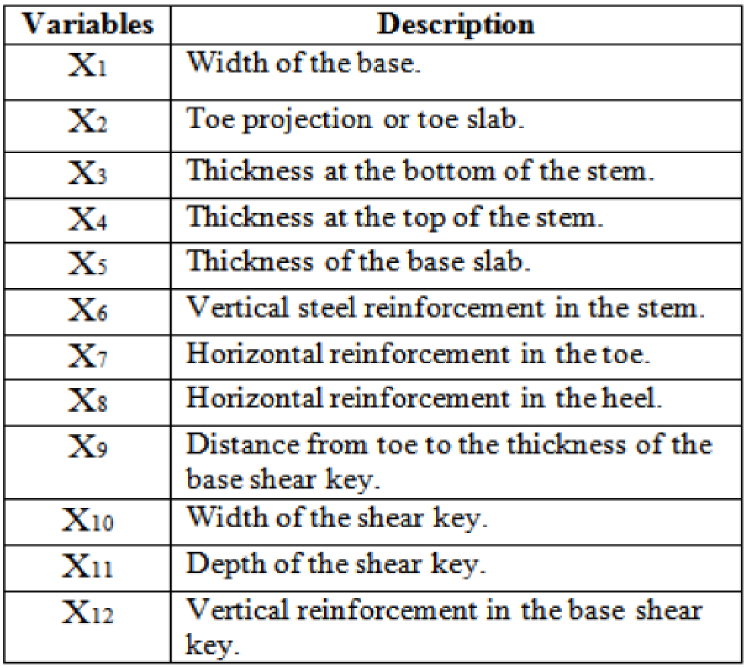
↳ Following are the inputs required for the optimizer:

- No of variables
- No of constraints.
- Maximum number of generations.
- Lower limit of variable.
- Upper limit of variables.

4.1 VARIABLES



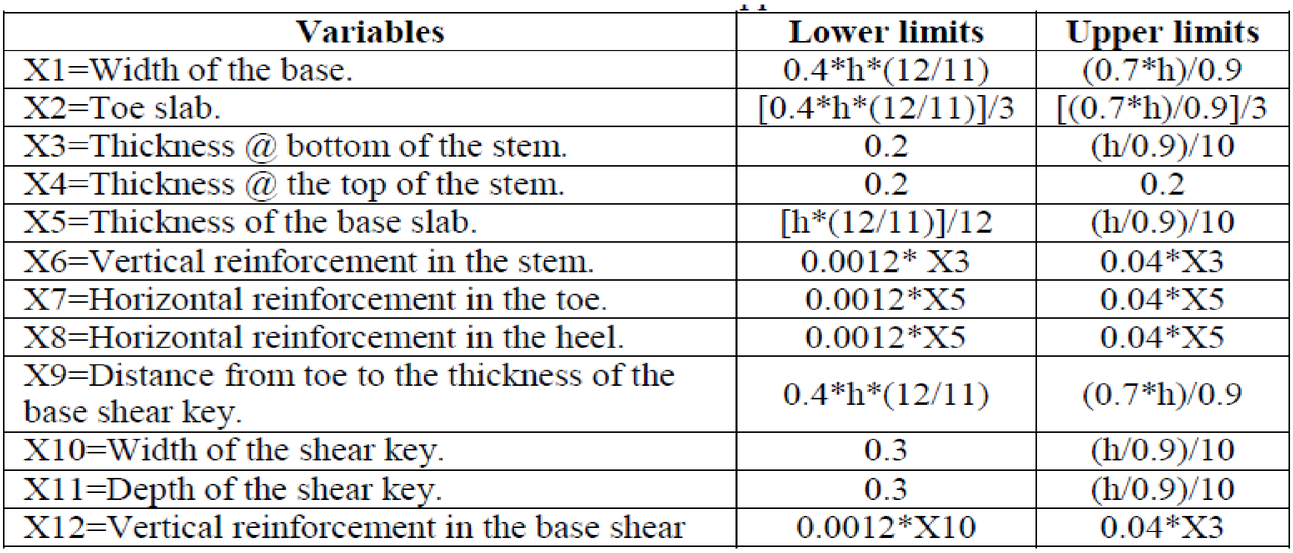
Variables diagram



Variables	Description
X_1	Width of the base.
X_2	Toe projection or toe slab.
X_3	Thickness at the bottom of the stem.
X_4	Thickness at the top of the stem.
X_5	Thickness of the base slab.
X_6	Vertical steel reinforcement in the stem.
X_7	Horizontal reinforcement in the toe.
X_8	Horizontal reinforcement in the heel.
X_9	Distance from toe to the thickness of the base shear key.
X_{10}	Width of the shear key.
X_{11}	Depth of the shear key.
X_{12}	Vertical reinforcement in the base shear key.

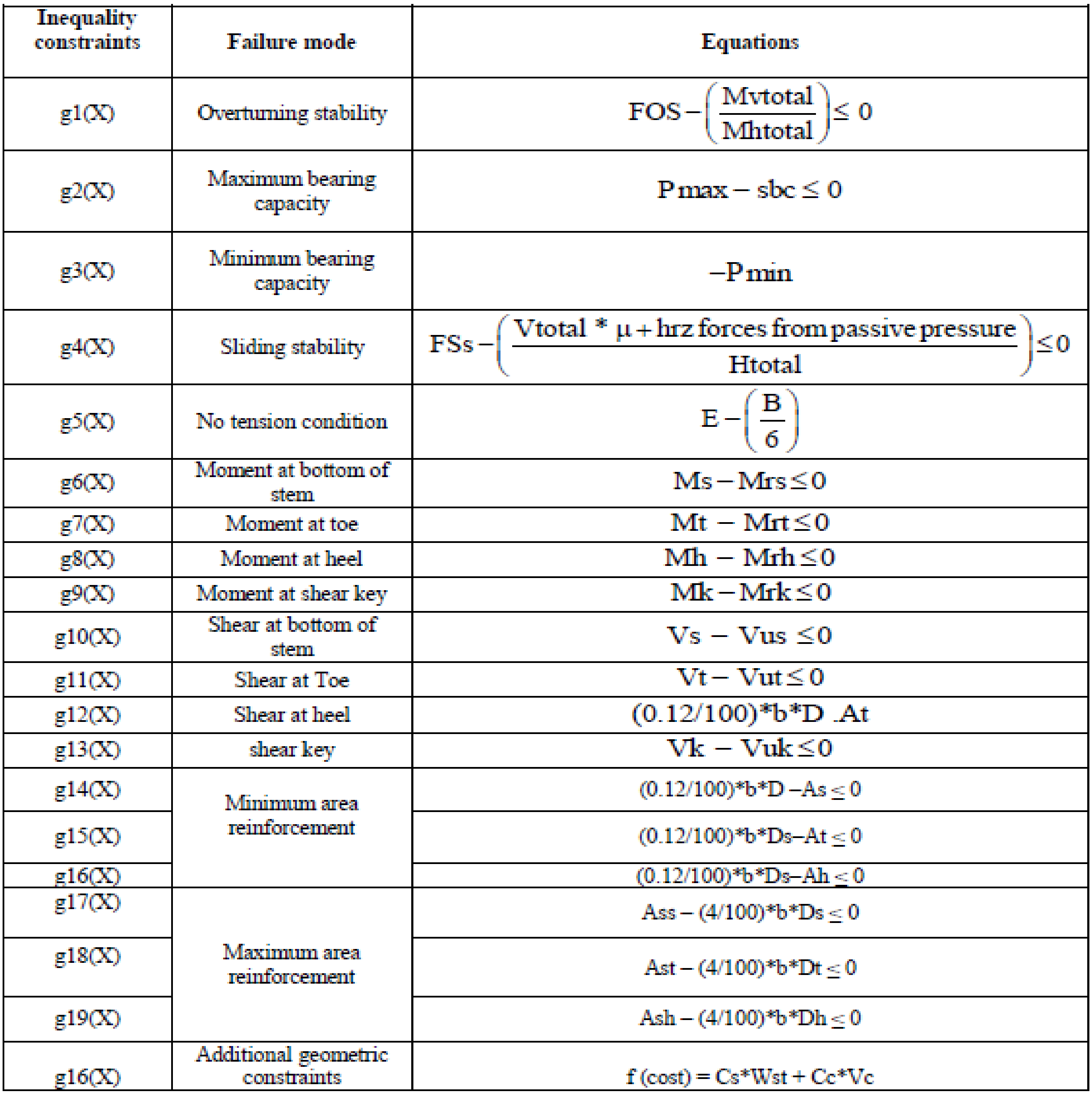
Variables diagram

4.2 Lower and upper limits



Variables	Lower limits	Upper limits
X1=Width of the base.	0.4*h*(12/11)	(0.7*h)/0.9
X2=Toe slab.	[0.4*h*(12/11)]/3	[(0.7*h)/0.9]/3
X3=Thickness @ bottom of the stem.	0.2	(h/0.9)/10
X4=Thickness @ the top of the stem.	0.2	0.2
X5=Thickness of the base slab.	[h*(12/11)]/12	(h/0.9)/10
X6=Vertical reinforcement in the stem.	0.0012* X3	0.04*X3
X7=Horizontal reinforcement in the toe.	0.0012*X5	0.04*X5
X8=Horizontal reinforcement in the heel.	0.0012*X5	0.04*X5
X9=Distance from toe to the thickness of the base shear key.	0.4*h*(12/11)	(0.7*h)/0.9
X10=Width of the shear key.	0.3	(h/0.9)/10
X11=Depth of the shear key.	0.3	(h/0.9)/10
X12=Vertical reinforcement in the base shear	0.0012*X10	0.04*X3

4.3 CONSTRAINTS



Inequality constraints	Failure mode	Equations
g1(X)	Overturning stability	$FOS-\left(\frac{Mvtotal}{Mhtotal}\right)\leq 0$
g2(X)	Maximum bearing capacity	$P\max - sbc \leq 0$
g3(X)	Minimum bearing capacity	$-P\min$
g4(X)	Sliding stability	$FSs-\left(\frac{Vtotal * \mu + \text{hrz forces from passive pressure}}{Htotal}\right)\leq 0$
g5(X)	No tension condition	$E-\left(\frac{B}{6}\right)$
g6(X)	Moment at bottom of stem	$Ms - Mrs \leq 0$
g7(X)	Moment at toe	$Mt - Mrt \leq 0$
g8(X)	Moment at heel	$Mh - Mrh \leq 0$
g9(X)	Moment at shear key	$Mk - Mrk \leq 0$
g10(X)	Shear at bottom of stem	$Vs - Vus \leq 0$
g11(X)	Shear at Toe	$Vt - Vut \leq 0$
g12(X)	Shear at heel	(0.12/100)*b*D .At
g13(X)	shear key	$Vk - Vuk \leq 0$
g14(X)	Minimum area reinforcement	$(0.12/100)*b*D - As \leq 0$
g15(X)		$(0.12/100)*b*Ds - At \leq 0$
g16(X)		$(0.12/100)*b*Ds - Ah \leq 0$
g17(X)	Maximum area reinforcement	$Ass - (4/100)*b*Ds \leq 0$
g18(X)		$Ast - (4/100)*b*Dt \leq 0$
g19(X)		$Ash - (4/100)*b*Dh \leq 0$
g16(X)	Additional geometric constraints	f (cost) = Cs*Wst + Cc*Vc

Nomenclature:

- Mhtotal = Total horizontal moment of forces that tends to overturn about toe.
  Mvtotal = Total vertical moment of forces that tends to resist overturning about toe.
- FSo = Factor of safety against overturning.
- Pmax=Maximum contact pressure at the interface between the wall structure and the foundation Soil. Pmin = Minimum contact pressure at the interface between the wall structure and the foundation soil. Htotal=Total horizontal driving forces. FSs=Factor of safety against sliding.
- B = Base width of the wall. E = Eccentricity of the resultant force.
- Mrs,Mrt,Mrh,Mrk = Flexural strength of stem, toe, heel, shear key.
- Ms,Mt,Mh,Mk= Maximum bending moment at the face of the wall, toe of slab, heel of slab, shear key. Vs, Vt, Vh, Vk = design shear carrying capacity at stem, toe, heel, shear key.
- Vus, Vut, Vuh, Vuk = Shear capacity of concrete at stem, toe, heel, shear key.
- As, At, Ah, Ak = Area of reinforcement at stem, toe, heel, shear key. ds,dt,dh,dk = Effective depth at stem, toe, heel, shear key.
- Cs is the unit cost of steel, Cc is the unit cost of concrete, Wst is the weight of steel per unit length of the wall, and Vc is the volume of concrete per unit.

The below table shows the rates for different grades of concrete and steel, as per Hubli- Dharwad schedule rates book.

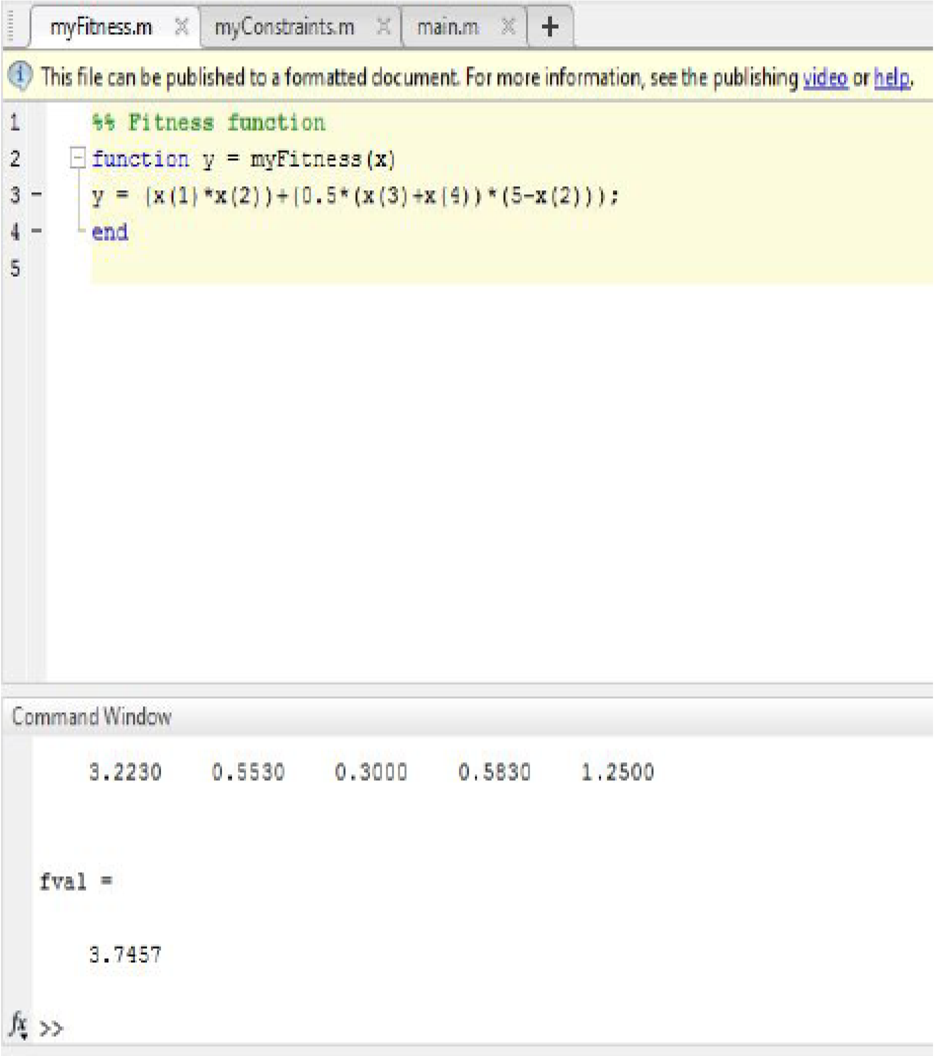
Grade of concrete in N/mm2	Rate in rupees per cubic meter
M20	□5200
M30	□6000
M40	□6600

Grade of steel in N/mm2	Rate in rupees per kilogram
415	□ 55
500	□ 60

4.4 Program for Algorithm

The program for algorithm to solve the created fitness function involves linking of fitness function, defining number of variables has to be defined, stating lower and upper bound values, linking of Constraint function and final code for algorithm. The program code for algorithm is given below.

```
[X, fval] = ga (objFcn, nvars, [ ], [ ], [ ], [ ], LB, UB, consFcn)
```



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```
%% Fitness function
function y = myFitness(x)
y = (x(1)*x(2))+(0.5*(x(3)+x(4))*(5-x(2)));
end
```

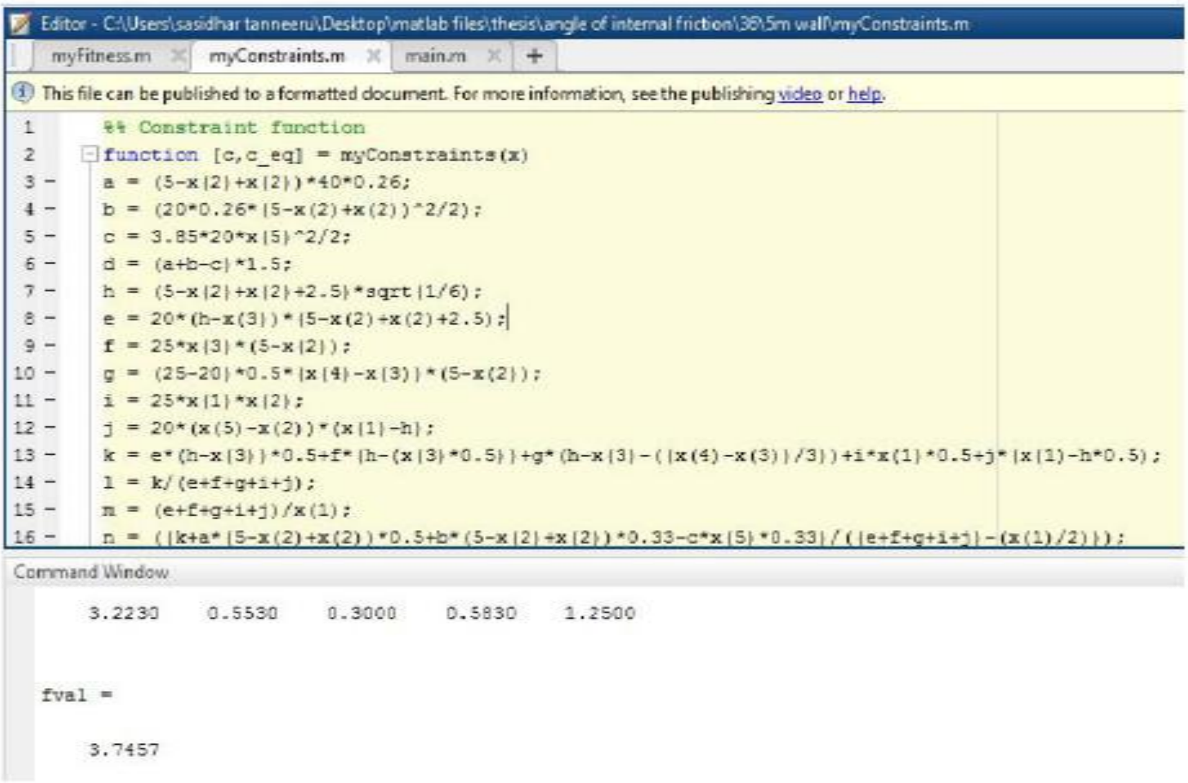
```
    3.2230    0.5530    0.3000    0.5830    1.2500


fval =

    3.7457

>>
```

Fitness function


Editor - C:\Users\sasidhar tanneeru\Desktop\matlab files\thesis\angle of internal friction\36\5m wall\myConstraints.m
myFitness.m
myConstraints.m
main.m
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Command Window

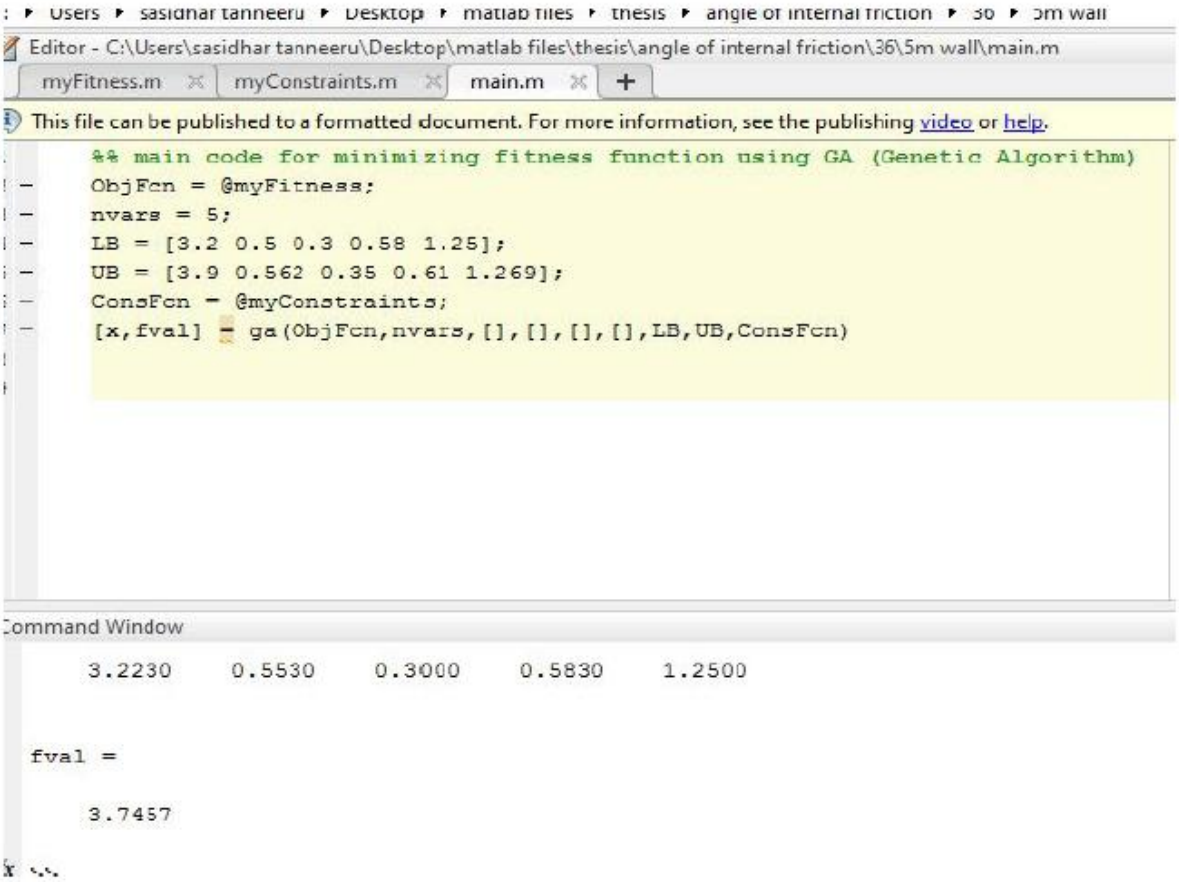
```
%% Constraint function
function [c,c_eq] = myConstraints(x)
a = (5-x(2)+x(2))*40*0.26;
b = (20*0.26*(5-x(2)+x(2))^2/2);
c = 3.85*20*x(5)^2/2;
d = (a+b-c)*1.5;
h = (5-x(2)+x(2)+2.5)*sqrt(1/6);
e = 20*(h-x(3))*(5-x(2)+x(2)+2.5);
f = 25*x(3)*(5-x(2));
g = (25-20)*0.5*(x(4)-x(3))*(5-x(2));
i = 25*x(1)*x(2);
j = 20*(x(5)-x(2))*(x(1)-h);
k = e*(h-x(3))*0.5+f*(h-(x(3)*0.5))+g*(h-x(3)-((x(4)-x(3))/3))+i*x(1)*0.5+j*(x(1)-h*0.5);
l = k/(e+f+g+i+j);
m = (e+f+g+i+j)/x(1);
n = ((k+a*(5-x(2)+x(2))*0.5+b*(5-x(2)+x(2))*0.33-c*x(5)*0.33)/((e+f+g+i+j)-(x(1)/2)));
```

```
    3.2230    0.5530    0.3000    0.5830    1.2500


fval =

    3.7457
```

Constraint function


Users ▸ sasidhar tanneeru ▸ Desktop ▸ matlab files ▸ thesis ▸ angle of internal friction ▸ 36 ▸ 5m wall
Editor - C:\Users\sasidhar tanneeru\Desktop\matlab files\thesis\angle of internal friction\36\5m wall\main.m
myFitness.m
myConstraints.m
main.m
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Command Window

```
%% main code for minimizing fitness function using GA (Genetic Algorithm)
ObjFcn = @myFitness;
nvars = 5;
LB = [3.2 0.5 0.3 0.58 1.25];
UB = [3.9 0.562 0.35 0.61 1.269];
ConsFcn = @myConstraints;
[x,fval] = ga(ObjFcn,nvars,[],[],[],[],LB,UB,ConsFcn)
```

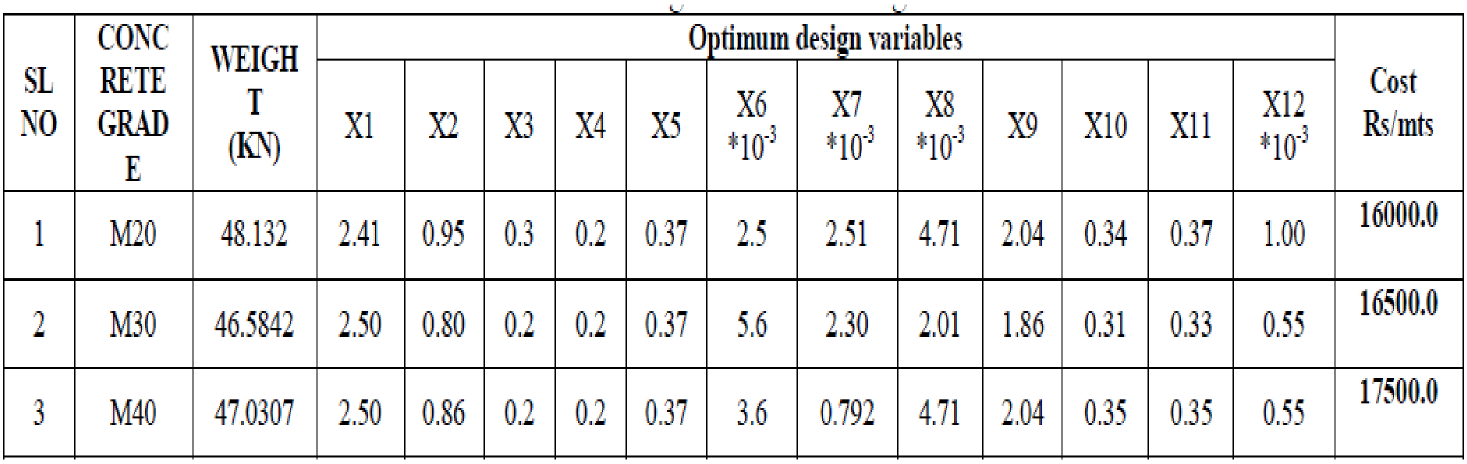
```
    3.2230    0.5530    0.3000    0.5830    1.2500


fval =

    3.7457
```

Program for genetic algorithm

5 Numerical example

A cantilever retaining wall to retain earth for a height of 4m. The backfill is horizontal. The density of soil is 18kN/m^3. S.B.C of soil is 200 kN/m^2. The co-efficient of friction between concrete and soil as 0.6. The angle of repose is 30°. Use M20 concrete and Fe415 steel.

1) Parametric studies on effect of change in cost of retaining wall for various grades of concrete for 4 mts height.

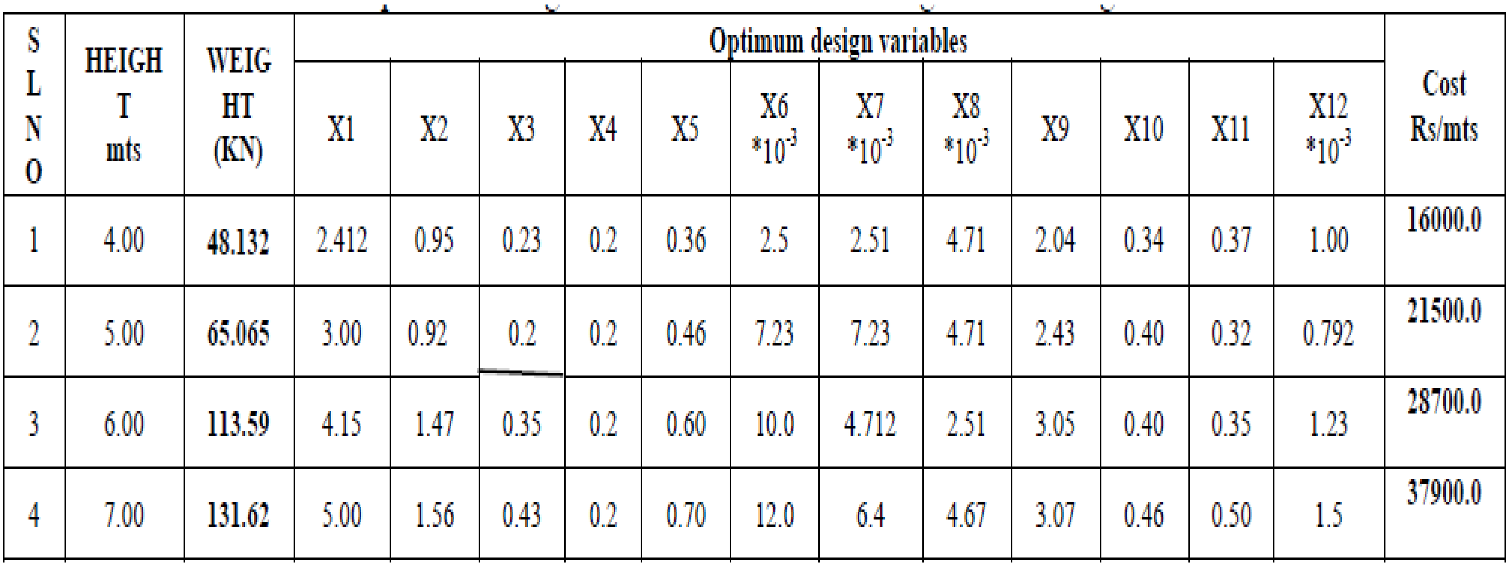


SL NO	CONCRETE GRADE	WEIGHT (KN)	Optimum design variables												Cost Rs/mts
			X1	X2	X3	X4	X5	X6 $*10^{-3}$	X7 $*10^{-3}$	X8 $*10^{-3}$	X9	X10	X11	X12 $*10^{-3}$	
1	M20	48.132	2.41	0.95	0.3	0.2	0.37	2.5	2.51	4.71	2.04	0.34	0.37	1.00	16000.0
2	M30	46.5842	2.50	0.80	0.2	0.2	0.37	5.6	2.30	2.01	1.86	0.31	0.33	0.55	16500.0
3	M40	47.0307	2.50	0.86	0.2	0.2	0.37	3.6	0.792	4.71	2.04	0.35	0.35	0.55	17500.0

Cost of retaining wall for various grades of concrete

- Weight of the retaining wall does not show significant changes for different grade of concrete.
- Note:
  - ❖ X1, X2, X3, X4, X5, X9, X10, X11, are geometric variable which are expressed in meters.
  - ❖ X6, X7, X8, X12, are reinforcement variable which are expressed in sq. meters.

2) Parametric studies on effect of change in retaining wall, heights for M20 grade of concrete.

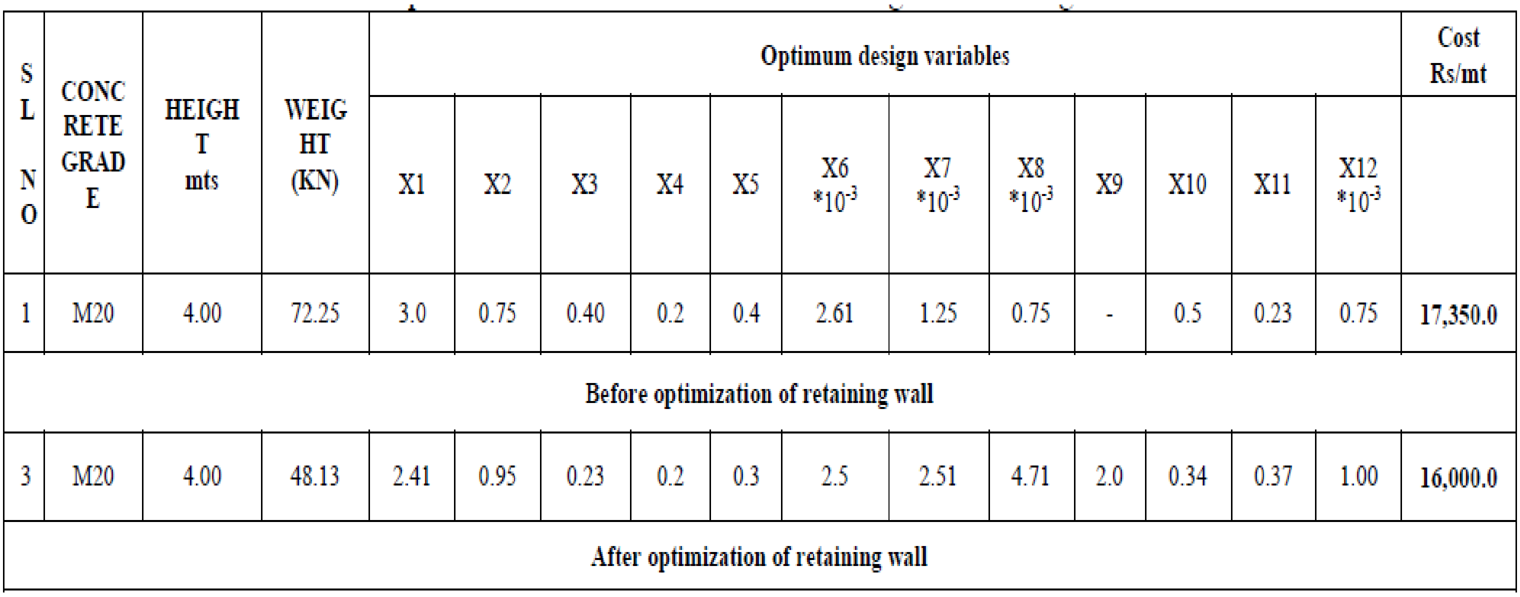


SL NO	HEIGHT mts	WEIGHT (KN)	Optimum design variables												Cost Rs/mts
			X1	X2	X3	X4	X5	X6 $*10^{-3}$	X7 $*10^{-3}$	X8 $*10^{-3}$	X9	X10	X11	X12 $*10^{-3}$	
1	4.00	48.132	2.412	0.95	0.23	0.2	0.36	2.5	2.51	4.71	2.04	0.34	0.37	1.00	16000.0
2	5.00	65.065	3.00	0.92	0.2	0.2	0.46	7.23	7.23	4.71	2.43	0.40	0.32	0.792	21500.0
3	6.00	113.59	4.15	1.47	0.35	0.2	0.60	10.0	4.712	2.51	3.05	0.40	0.35	1.23	28700.0
4	7.00	131.62	5.00	1.56	0.43	0.2	0.70	12.0	6.4	4.67	3.07	0.46	0.50	1.5	37900.0

Optimum design variables for different heights for M20 grade concrete

- Weight to height ratio is 12.03, 13.013, 18.9, and 18.8.
- Note:
  - ❖ Parameters such as SBC= 200 kN/m2, γ= 18 kN/m3, µ=0.6, ϕ=30°.

3) Optimum cost and variables of retaining wall.



SL NO	CONCRETE GRADE	HEIGHT mts	WEIGHT (KN)	Optimum design variables												Cost Rs/mt
				X1	X2	X3	X4	X5	X6 $*10^{-3}$	X7 $*10^{-3}$	X8 $*10^{-3}$	X9	X10	X11	X12 $*10^{-3}$	
1	M20	4.00	72.25	3.0	0.75	0.40	0.2	0.4	2.61	1.25	0.75	-	0.5	0.23	0.75	17,350.0
Before optimization of retaining wall																
3	M20	4.00	48.13	2.41	0.95	0.23	0.2	0.3	2.5	2.51	4.71	2.0	0.34	0.37	1.00	16,000.0
After optimization of retaining wall																

Optimum cost and variables for 4 m height and M20 grade of concrete.

- Note:
  - ❖ Parameters such asSBC= 200 kN/m2, γ= 18 kN/m3, µ=0.6, ϕ=30°.

Comment :

- Genetic algorithm can be satisfactorily applied for the optimization of cantilever retaining wall.
- There is sudden change is observed for weight and height ratio, between 5mts and 6mts
- Weight of the retaining wall has been reduced from 72.25 KN to 48.13KN.
- Some geometric parameters of retaining wall have been reduced and some are increased.
- Cost of retaining wall has reduced from 17,350 to 16,000 Rs/mts of length.

6 CONCLUSION

Use of genetic algorithm technique in the design of retaining wall has given considerable percentage of cost optimization in all the cases which are taken for the analysis. This genetic algorithm technique is found to be useful in the designs where number of trails have to be done for making it more economical. It requires lower and upper bound values of considered variables. The range of boundary conditions will alter the objective function value. Hence, the provision of genetic algorithm technique in the design of retaining wall is advantageous in optimization of cost. But it is time taking method as it requires new program for every change properties of retaining wall. There is more scope to work on this modern optimization technique in the design of structural elements.

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